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# Analysis of Garofalo equation parameters for an ultrahigh carbon steel

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**Abstract** Isothermal stress–strain curves data from torsion tests conducted at high temperature (950–1200 °C) and strain rates (2–26 $s^{-1}$) were analyzed in an ultrahigh carbon steel (UHCS) containing 1.3%C. The sine hyperbolic Garofalo equation was selected as an adequate constitutive equation for the entire range of the forming variables considered. The Garofalo parameters were assumed strain dependent allowing the prediction of stress–strain curves under transient and steady-state conditions. The average relative errors obtained were below 3% in stress. In addition, the creep deformation mechanisms in the UHCS were analyzed from the Garofalo equation parameters. For this aim, the stress exponent of the Garofalo equation was, for the first time, related to that of the power law equation. The results show that the controlled deformation mechanism at steady state is lattice diffusion-controlled slip creep.

## Introduction

Torsion tests at various temperatures and strain rates are usually carried out in order to evaluate the hot deformation behavior of materials. These tests are especially adequate for this aim since they allow high deformations minimizing plastic instability [1].

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Torsion tests provide a set of data of stress, $\sigma$, strain rate, $\dot{\varepsilon}$, and temperature, $T$, These variables are often related, at steady state, by means of a power law creep equation [2, 3]

$$\dot{\varepsilon} = A \exp\left(-\frac{Q}{RT}\right)\sigma^{n_p} \quad (1)$$

where $A$ is a constant, $Q$ is the activation energy for creep, $R$ is the gas constant, and $n_p$ is the stress exponent. A large number of materials have been investigated using this equation with a value of five for the stress exponent in order to describe the creep behavior at steady-state deformation [4, 5].

In addition, the sine hyperbolic equation, or Garofalo equation, is also employed for describing the creep behavior of materials in a wider range of temperatures and strain rates [6, 7]

$$\dot{\varepsilon} = A \exp\left(-\frac{Q}{RT}\right)\sinh^{n_g}(\alpha\sigma) \quad (2)$$

where $A$, $Q$, $\alpha$, and $n_g$ are material parameters. Usually, this equation is fitted at the strain corresponding to the peak stress or at the strain where the steady state is reached. In contrast, recent works fit the Garofalo equation at various levels of strain predicting the creep behavior under transient and steady state deformation [8–10].

As mentioned, the advantage of the Garofalo equation lies in its capability to predict the creep behavior of materials under very wide range of temperatures and strain rates. For this reason, the Garofalo equation is nowadays more and more employed to describe the flow behavior of materials. However, the association of the Garofalo equation parameters with usual deformation mechanisms at high temperature is different from that involving the power law equation. Apart from $\alpha$, which is not included in the power law equation, only the activation energy has been

shown to be the same for both equations [4, 11]. In contrast, the difference between $n_p$ and $n_g$ is usually large [8]. It should be mentioned that the relation between the parameters from both Eqs. 1 and 2, is poor and is only based on statistical correlations [6].

The objective of this work is threefold: (a) to establish a physical based relation between $n_p$ and $n_g$, (b) to determine the creep deformation mechanism operating in an ultrahigh carbon steel (UHCS) containing 1.3%C (UHCS-1.3%C) using the relation for the evolution of the Garofalo equations parameters with strain and the relation established in (a), and (c) to predict the stress–strain behavior under transient and steady state conditions for this steel.

## Material and experimental details

The UHCS studied in this investigation has the following composition: 1.3%C, 0.5% Mn, 0.6% Si, 0.18% Cr, and balance Fe. The manganese was added to neutralize the deleterious effects of sulfur and phosphorus. The steel was produced at Sidenor Company as a cast of 8 l by means of an induction furnace. The as-cast ingot was initially soaked at 1050 °C and forged into a bar of 60 mm × 55 mm cross section.

Torsion tests were conducted in a SETARAM torsion machine at CENIM in the temperature range 950–1200 °C at strain rates in the range 2–26 $s^{-1}$. An induction furnace heats the test sample and the temperature is continuously measured by means of a two-color pyrometer. A silica tube with argon atmosphere ensures protection against oxidation. A helium atmosphere is used to obtain, after testing, a cooling rate of 325 K/s. The torsion samples have an effective gage length of 17 mm and a radius of 3 mm.

## Results and discussion

### Stress–strain behavior

Torsion tests at various temperatures and strain rates were conducted in an UHC-1.3%C steel. For each torsion test, data of torque and number of turns were converted to shear stress and shear strain [12]. The von Mises method was selected to obtain the values of stress, strain, and strain rate. Moreover, adiabatic heating usually occurring during high strain rates torsion tests were also considered in order to obtain stress–strain curves at constant temperature and strain rate. It is worth noting that adiabatic heating strongly affects the shape of the stress–strain curves as well as the absolute values of the Garofalo equation parameters. The correction of adiabatic heating was conducted by a new algorithm described elsewhere [13]. The algorithm is based on two equations. The first one is the usual equation for the temperature rise with strain of the bulk material during deformation, $\Delta T$

$$\Delta T = \frac{\eta}{\rho C}\int_0^{\varepsilon} \sigma_0^{wc}(s)\mathrm{d}s \tag{3}$$

where $\eta$ is the Taylor–Quinney factor, $\rho$ is the density, $C$ is the specific heat, and $\sigma_0^{wc}$ is the stress without adiabatic heating correction. The second equation allows the calculation of the stress under isothermal conditions, $\sigma_0^c$, which is corrected from the one under adiabatic conditions.

$$\sigma_0^c(\varepsilon,\dot{\varepsilon},T) = \sigma_0^{wc}(\varepsilon,\dot{\varepsilon},T) - \left.\frac{\partial\sigma_0^{wc}(T)}{\partial T}\right|_{\varepsilon,\dot{\varepsilon}} \Delta T \tag{4}$$

The calculations were performed assuming $\eta = 0.9$ for torsion samples with aspect ratio 5.7:1. The initial and final torsion test temperatures were monitored. An iterative algorithm was implemented for determination of the temperature increment, which serves as basis of the stress correction. The iterative process was stopped for a temperature variation of less than 1 K.

Figure 1 shows the isothermal stress–strain behavior of the UHCS at all temperatures and strain rates. The curves are characterized by a continuous increase of the stress to a peak value following by a decrease to a steady state previous to rupture. The beginning of a steady-state regime can be identified at strains of about 2. This behavior is usually attributed to the presence of dynamic recrystallization being the main softening phenomena [14, 15].

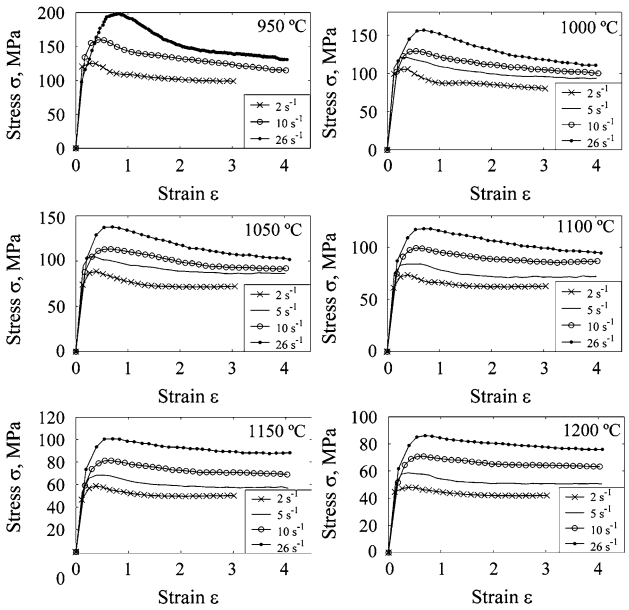


**Fig. 1** Isothermal stress–strain behavior of the UHCS-1.3%C

## Modeling of stress–strain behavior

The Garofalo equation, Eq. 2, was used to model the stress–strain curves of the UHCS-1.3%C. The equation is fitted to $\{\dot{\varepsilon}, \sigma, T\}$ data at various levels of strain in order to predict the transient and steady-state regimes of deformation [8]. Concretely, these fittings are based on 23 torsion tests and the Garofalo parameters are determined at strains of 0.4, 0.5, peak (about 0.54), 0.7, 0.8, 1, 1.5, 2, 2.5, 3, and 3.5. This large selection of strains allowed us to accurately compare results from both Eqs. 1 and 2, with very different strain ranges of application. It should be noted that we chose a Taylor–Quinney factor of 0.9 which means that 90% is the maximum value of the stored energy that can be attributed to adiabatic heating. This is a high bound for steels according to literature [16, 17]. Furthermore, the softening cannot be attributed to a texture effect through a decrease of Taylor factor since rather low intensity textures are obtained in UHCS mainly because the large amount of carbides present. For these reasons, we could consider that all the softening measured after adiabatic heating correction is due to dynamic recrystallization. The determination of the Garofalo parameters is carried out by means of the Rieiro–Carsí–Ruano, RCR, method described elsewhere [18]. In contrast to the traditional methods, the RCR method is an improvement without the use of initial values providing, additionally, statistical parameters to quantify the goodness of the fit.

Table 1 shows the Garofalo equation parameters as a function of strain. The last two columns show the $R^2$ and $F$-Snedecor statistical parameters. The high $F$- and $R$-values are an indication of high goodness of the fit. Each parameter characterizes different aspects of the goodness of the fit of the Garofalo equation to the experimental data set [18]. For strains lower than 0.4 the quality of the fits are low and are not given in the table. This is attributed to spurious elastoplastic effects occurring at these high strain rates.

**Table 1** Garofalo equation and statistical parameters as a function of strain for the UHCS-1.3%C

| $\varepsilon$ | $n_g$ | $Q$ (kJ/mol) | $\alpha$ (MPa$^{-1}$) | ln($A$), $A$ in s$^{-1}$ | $R^2$ | $F$ |
|---|---|---|---|---|---|---|
| 0.4 | 3.66 | 324.83 | 0.0163 | 27.61 | 0.96 | 237.2 |
| 0.5 | 3.96 | 273.01 | 0.0103 | 25.45 | 0.99 | 850.3 |
| Max.(~0.54) | 4.31 | 270.12 | 0.0073 | 27.00 | 0.97 | 369.7 |
| 0.7 | 3.78 | 231.20 | 0.0076 | 23.34 | 0.99 | 787.5 |
| 0.8 | 3.99 | 223.69 | 0.0053 | 24.42 | 0.98 | 652.1 |
| 1 | 3.77 | 212.16 | 0.0063 | 22.66 | 0.98 | 434.8 |
| 1.5 | 3.53 | 211.55 | 0.0090 | 21.22 | 0.97 | 384.2 |
| 2 | 2.85 | 218.43 | 0.0173 | 19.24 | 0.98 | 651.2 |
| 2.5 | 2.7 | 223.12 | 0.0210 | 18.96 | 0.98 | 429.2 |
| 3 | 2.51 | 225.35 | 0.0246 | 18.68 | 0.97 | 376.3 |
| 3.5 | 2.47 | 226.76 | 0.0267 | 18.45 | 0.96 | 223.5 |

Once the Garofalo equation parameters are determined the stress at the strains previously selected can be predicted. For this aim, the Garofalo equation can be written as follows:

$$\sigma = \frac{1}{\alpha^{\varepsilon}} \sinh^{-1}\left(Z_A^{\varepsilon}\right) \tag{5}$$

where the superscript $\varepsilon$ indicates a dependence with strain and

$$Z_A^{\varepsilon} = \left(\frac{Z^{\varepsilon}}{A^{\varepsilon}}\right)^{\frac{1}{n_g^{\varepsilon}}} \tag{6}$$

being $Z^{\varepsilon}$ the Zener–Hollomon parameter, $Z^{\varepsilon} = \dot{\varepsilon}\exp(Q^{\varepsilon}/RT)$.

Figure 2 shows the comparison between the experimental and predicted stress from Eq. 5. Except for the strain 0.4, and as expected from the high values of the statistical parameters given in the last two columns of Table 1, the average relative errors between the experimental and predicted stress are lower than 3.5%. These errors are similar or even lower than other recently presented in the literature [19, 20]. Figure 2 confirms that Eq. 5, in average terms, may predict the stress under transient and steady state conditions. These results allow us to conclude that the Garofalo equation can be considered as a suitable constitutive equation for the UHCS-1.3%C under transient and steady state creep regime [8].

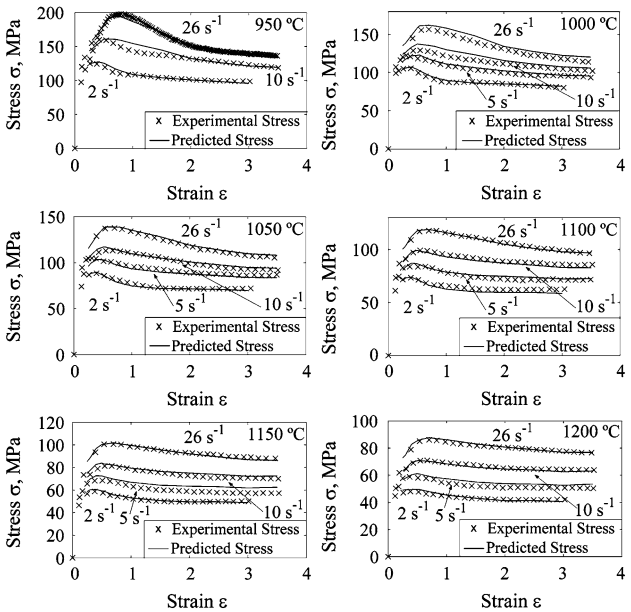


**Fig. 2** Comparison between predicted and experimental stress for the UHCS-1.3%C. The predicted stress is given by Eq. 5

## Relation between stress exponents and analysis of deformation mechanism

Figure 3 shows the Garofalo equation parameters as a function of strain in the interval from 0.54 (average value of peak strain) to 3.5. Except for the α parameter, a clear steady state regime is reveled for strains higher than 2, which is in agreement with the results obtained for the stress–strain curves. It is also observed that while $n$ continuously decreases with strain, α continuously increases with strain. This behavior was previously reported by McQueen and Ryan as a general behavior in austenitic stainless steels [6].

According to Fig. 3, $n_g$ is about 2.5 at the higher strains, a value that is quite lower than 5, which is the expected one for a large number of alloys when the power law is employed as constitutive equation [4]. An interpretation of this value of 2.5 as associated to a governing mechanism other than slip creep would be erroneous. However, this difference can be explained by understanding the physical meaning of both stress exponents. A new relation between the stress exponents of the power law and the Garofalo equations, $n_p$ and $n_g$, has been developed for this aim. The stress exponent corresponding to the power law equation can be expressed as:

$$n_p = \left.\frac{\partial \ln(\dot{\varepsilon})}{\partial \ln(\sigma)}\right|_T \tag{7}$$

and according to the Garofalo equation

$$n_g = \left.\frac{\partial \ln(\dot{\varepsilon})}{\partial \ln(\sinh(\alpha\sigma))}\right|_T \tag{8}$$

Applying the chain rule to Eq. 8 leads to

$$n_g = \left.\frac{\tanh(\alpha\sigma)}{\alpha\sigma}\frac{\partial \ln(\dot{\varepsilon})}{\partial \ln(\sigma)}\right|_T \tag{9}$$

The following relation between the $n_p$ and $n_g$ parameters is obtained from Eqs. 7 and 9

$$n_p = \left.\frac{\alpha\sigma}{\tanh(\alpha\sigma)} n_g\right|_T \tag{10}$$

This relation can be rewritten as follows:

$$n_p^{\varepsilon} = \left.\frac{\alpha^{\varepsilon}\sigma^{\varepsilon}}{\tanh(\alpha^{\varepsilon}\sigma^{\varepsilon})} n_g^{\varepsilon}\right|_T \tag{11}$$

This relation allows converting, for a given strain, stress exponents obtained by the fit of the Garofalo equation into stress exponents from the power law equation. Assuming $n_p$ as a constant for a given temperature and strain, Eq. 11 shows that $n_g$ decreases with increasing α as supported experimentally [6]. The equation also reveals that the usual identification of the Garofalo equation stress exponent with the inverse of the strain rate sensitivity, $m$, is only valid in the range where $\alpha\sigma < 1$ since $\alpha\sigma/\tanh(\alpha\sigma)$ approaches to 1. The relation $m = 1/n$ is only fulfilled at low stresses where $n_g \cong n_p$.

Equation 11 can be applied at every temperature and strain in order to check the dependence of $n_p$ with strain and temperature. Figure 4 shows the evolution of the calculated $n_p$, obtained by Eq. 11, with temperature. The points in the figure correspond to strains of 2, 2.5, and 3 and are obtained from experimental data. The figure shows that small differences in $n_p$ values are present at the highest temperatures, which is an evidence of steady state. In addition, Fig. 4 shows that the $n_p$ parameter, for a given strain, is temperature dependent revealing a variation from values of about 7.4 at 950 °C to 4 at 1200 °C. This result shows that an explicit temperature dependence of the $n_p$ parameter should have been incorporated in the power law equation in order to describe the creep data of the UHCS-1.3%C. This was previously theoretically advanced in [21].

Finally, in order to compare the values of the $n_p$ parameter with those of the stress exponent of the Garofalo equation, $n_g$, an average value over temperature of $n_p$, $n_p^{av}$, can be computed at each strain. These values are

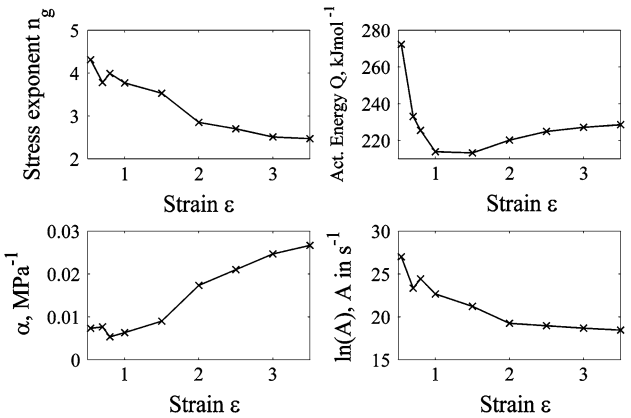


**Fig. 3** Garofalo equation parameters as a function of strain for the UHCS-1.3%C

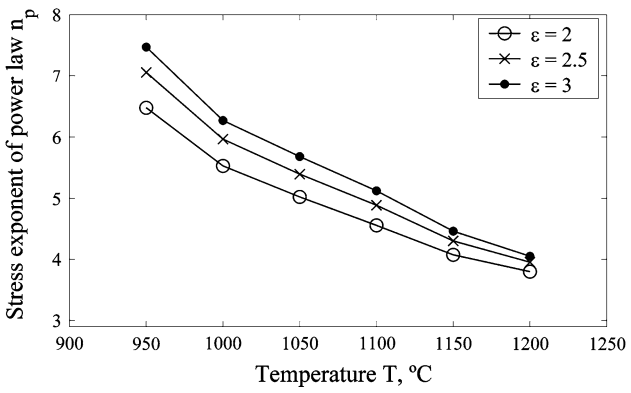


**Fig. 4** Variation of the stress exponent of the power law, $n_p$, with temperature for strains of 2, 2.5, and 3 (steady state). Data are obtained from Eq. 11

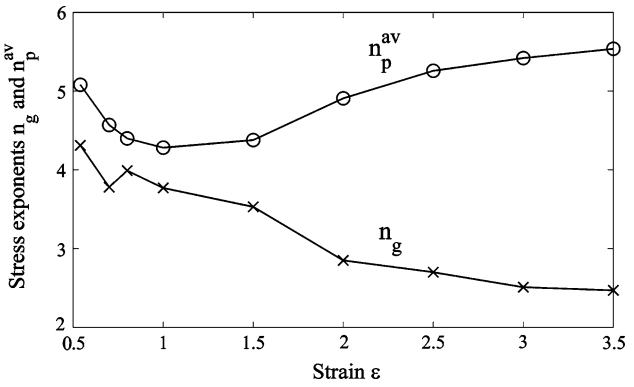


**Fig. 5** Comparison between the Garofalo stress exponent, $n_g$, and the average stress exponent of the power law $n_p^{av}$ as a function of strain

represented as a function of strain in Fig. 5. All points in the figure correspond to experimental data. It is observed that the $n_g$ parameter slightly decreases with strain. In contrast, $n_p^{av}$ slightly increases with strain with a value of about 5 at steady state (strains from 2 to 3) and for the strain at peak ($\sim$0.54). In other words, the $n_p^{av}$ values given in the figure is the result of two components, $n_g$ and $\alpha$, from the Garofalo equation. The evolution of these components with strain results in a value of $n_p^{av}$ of about 5 for large strain ranges where the power law is valid. These results seem to indicate that, in average terms, the creep behavior of the UHCS-1.3%C responds to a five-power-law equation. The high correlation obtained for $n_g$ values at strains between 2 and 3 allows concluding that the five power law in the steady state corresponds to a stress exponent of about 2.5 for the Garofalo equation. Finally, it should be noted that the comparison of $n_p$ and $n_g$ values should be performed through the average values of $n_p$ since $n_g$ is an average value by itself calculated over all temperatures of the test.

On the other hand, a high activation energy is computed at peak, 270 kJ/mol. This value is much higher than that corresponding to the self-diffusivity of iron in the austenite for the UHCS, 225 kJ mol$^{-1}$ [22]. This high value is attributed to the microstructure that is strongly changing since it is affected by dynamic recrystallization starting before the peak stress is reached. In contrast, at steady state regime, i.e., at strains higher than 2, the activation energy is almost constant with an average value of 224 kJ/mol that agrees well with the self-diffusivity of iron in the austenite. This confirms that the controlling deformation mechanism at steady state is lattice diffusion-controlled slip creep. In addition, it confirms that the activation energy obtained at the peak strain varies from that obtained at the steady state.

It is worth noting that the power law equation has a physical meaning and it is easy to optimize. On the other hand, the Garofalo equation is mathematically complex and difficult to optimize as described previously [18]. It has been proven that the power law is a first order approximation of Garofalo equation. For this reason, the parameters are not all directly related. For example, the parameter $n$ is not the same in both equations. This has a consequence in plastic deformation modeling because the exponent $n_p$ of the power law is associated with a specific deformation mechanism. Therefore, for a given deformation mechanism, it is necessary to have an equivalence between the parameter of the power law equation and that of the Garofalo equation. This has been done in this work for an UHCS.

## Conclusions

1. The Garofalo equation can be considered as a suitable constitutive equation for the UHCS-1.3%C at steady state and at the various regimes that characterize the stress–strain curve. The average relative errors between the predicted stress and the experimental data are lower than 3.5%.
2. A relation between the stress exponents of the Garofalo, $n_g$, and power-law, $n_p$, equations has been established. This relation predicts an increase of $n_g$ with a decrease of $\alpha$. The usual identification of the stress exponent of the Garofalo equation as the inverse of the strain rate sensitivity is only valid for $\alpha\sigma < 1$.
3. The value of $n_g$ of about 2.5 for the UHCS-1.3%C is similar to a value of about 5 for the stress exponent of the power law.
4. The controlling deformation mechanism is consistent with lattice diffusion-controlled slip creep.

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